United States Naval Postgraduate School

THESIS

AN ADAPTIVE DECISION PROCESS

by

Joseph Maris Moroney

December 1969

33

An Adaptive Decision Process

by

Joseph Maris Moroney
Lieutenant, United States Navy
B.S.E.E., University of Washington, 1962

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1969

Author ______________________

Approved by ______________________
Thesis Advisor

____
Chairman, Department of Operations Analysis

____
Academic Dean

ABSTRACT

This thesis investigates a decision process which is designed to employ multiple observations in an optimal manner as a means of solving a signal detection problem. The characteristic of this decision process is that it permits the decision-maker to defer his detection decision until new data is obtained from the next observation and to weigh the new data with an opinion based on previous data. The effect of adapting a decision to the results of previous observations is seen to be similar to a learning process which is taking place over a length of time. Since the decision process may involve relatively lengthy periods of time an estimator of this time is developed. Lastly, the decision model is seen to provide a model by which human detection behavior may be evaluated.

TABLE OF CONTENTS

I. AN ADAPTIVE DECISION PROCESS

An integral part of a signal detection process is the application of decision theory methods to the problem of determining whether or not the signal of interest is present. The techniques of decision theory assist the decision-maker in making a detection decision in the presence of random noise, with minimum error.

Classical detection is concerned with deciding the presence or absence of a useful signal in the presence of random noise after a single observation of fixed length has been made in a signal environment. The detection process may be complicated by a requirement to detect a signal with unknown parameters. This thesis will investigate a procedure by which multiple observations of a signal environment will assist in detecting such a signal.

The emphasis of this thesis will be on the discussion of a decision process which is designed to make use of multiple observations and on the application of this decision process to the problem of detecting an electromagnetic signal with known characteristics and transmitted on an unknown frequency. The decision process by which this is accomplished is shown to have optimum properties which may be applied to other detection problems involving multiple observations.

The decision process by which the detection of a signal with an unknown parameter is accomplished will be referred to as an <u>adaptive decision process</u>. The characteristic of this process is that it permits the decision-maker to defer his decision until new data is obtained and to weigh the new data with an opinion based on previous data.

PRECEDING PAGE BLANK

II. STATEMENT OF THE PROBLEM

The problem which is to be investigated in this thesis is the application of an adaptive decision process to detect a useful signal in the presence of random noise when a parameter of the signal is unknown. The process employs a scanning procedure in which a detection device may take repeated "looks" at a set of locations in which the useful signal is suspected to be present. The objective of the scanning procedure is to arrive at a state in which the unknown parameter is known and the useful signal is detected or conversely be in a state in which it is known that the useful signal is not present. Further, the decision process has the capability to make use of information derived from the preceding k-1 scans when the k^{th} scan has been completed.

The nature of the decision process will require that the useful signal have certain minimal characteristics:

1. The useful signal must exist long enough so that a minimal number of scans may be made. (The minimal number of scans may be computed; this topic will be discussed later.)
2. An a priori probability can be assigned to the event that the signal of interest is present in the set of locations being scanned.
3. Only one useful signal is present in the set of locations being scanned.

III. BACKGROUND

Classical detection theory is concerned with deciding the presence or absence of a useful signal in the presence of random noise after a single observation of fixed length has been made in a signal environment. The goal of this procedure is to make a detection decision with minimum error. The detection error is due to the noise which tends to mask the useful signal in a random manner. As a result, "guessing" is required to determine the presence of a useful signal; thus one signal may be confused with another or may go entirely undetected. The random nature of the background noise and the dependence of the useful signal on random parameters suggests decision theory methods should be employed in resolving the detection problem.

A. DECISION RULES

The decision theory approach to the detection problem begins with the formulation of the input to the detection process as the union of the useful signal $m(t)$ and the noise $n(t)$, given as functions of time,

$$f(t) = m(t) + n(t) \quad .$$

Next, it is necessary to compute the *a posteriori* probability that the signal $m(t)$ is present in the received function $f(t) = m(t) + n(t)$.

The *a posteriori* probability $P(m|f)$ that the useful signal $m(t)$ is present, given an input signal $f(t)$, can be expressed as

$$P(m|f) = \frac{P(m)P(f|m)}{P(f)} \quad .$$

Further, $f(t)$ is a compound event so $P(f)$ may be expressed as

$$P(f) = P(m)P(f|m) + P(n)P(f|n) \text{ , and}$$

$$P(m) + P(n) = 1 \text{ , where}$$

$P(m)$ is the <u>a priori</u> probability that the useful signal is present and $P(n)$ is the <u>a priori</u> probability that the useful signal is absent. After substituting for $P(f)$ we have

$$P(m|f) = \frac{P(m)P(f|m)}{P(m)P(f|m) + P(n)P(f|n)} = \frac{L}{L + [P(n)/P(m)]} \quad .$$

The quantity <u>L</u> is the <u>likelihood ratio</u> and is defined as

$$L = \frac{P(f|m)}{P(f|n)} \quad .$$

The <u>a posteriori</u> probability that the useful signal is absent may be expressed as

$$P(n|f) = \frac{P(n)/P(m)}{L + [P(n)/P(m)]} \quad .$$

Lastly, the ratio of the <u>a posteriori</u> probabilities equals

$$\frac{P(m|f)}{P(n|f)} = \frac{P(m)}{P(n)} L \quad .$$

Next, on the basis of the input signal received by the detector and its likelihood ratio (L), a signal detection decision may be made according to a decision rule, such as the following rule,

If $L \geq L^*$, the signal is present;

If $L_* < L < L^*$, no decision is made;

If $L \leq L_*$, the signal is absent.

The terms L_* and L^* are referred to as thresholds; their values are determined by the detection error which is allowable in the detection process. This decision rule will be of interest in later discussions. This decision process is called a sequential process because of the manner in which input data is handled. For each

observation, of a signal environment for example, a likelihood ratio (L) is computed in the manner described above; this L is compared to L_* and L^*, if $L_* < L < L^*$ no detection decision is made and another observation is made. This process continues until L exceeds L_* or L^*. It is well known that with probability 1 the sequential process will stop after a finite number of observations at which time the threshold L_* or L^* will have been reached.

In previous discussion of the detection process, it was noted that the random effects of the noise portion of the input signal could cause errors in the detection decision. Two kinds of errors are possible:

1. The false alarm error which results from interpreting noise to be the sum of signal and noise; the false alarm probability is denoted by F.
2. The false dismissal error which results from interpreting the sum of signal plus noise to be just noise; the false dismissal probability is denoted by 1-D, where D is the probability of a correct detection.

The values of the thresholds L_* and L^* will be defined as

$$L^* = \frac{1-D}{1-F}\ , \text{ and}$$

$$L_* = F/D\ .$$

Lastly, if the concept of costs of errors is considered, the optimum decision rule is the rule with the smallest cost. Let C_{1-D} be the cost of a false dismissal and C_F be the cost of a false alarm; the costs of correct detection and correct dismissal may be taken to be zero. If the observer pays the cost corresponding to

which of the two errors occurred his expected loss or risk is

$$\text{Risk} = P(m)(1-D)C_{1-D} + P(n)FC_F \ .$$

B. ADAPTIVE DECISION PROCESS

A large number of detection problems exist in which the detection decision process is complicated by the requirement to detect the presence of a useful signal which has unknown parameters. Fralick [1] proposes an adaptive decision process to handle this type of detection problem.

The adaptive decision process employs a decision rule which allows the decision maker to defer a detection decision in a manner somewhat similar to that discussed in the sequential decision process. The primary difference is that the adaptive decision process accumulates information through a recursive process in which the detector might be considered to learn the nature of the unknown parameter; the sequential decision process merely takes another independent observation then computes and compares a new likelihood ratio, based on that observation, to the thresholds.

IV. ADAPTIVE DECISION PROCESS MODEL

For convenience, the input to the detector in this model will be denoted by the symbol $f_{k+1}(t)$ as a way of indicating this particular input is being considered during the interval $kT \le t \le (k+1)T$.

From development of a likelihood ratio in preceding discussion, we may state that

$$L(f_{k+1}) = \frac{P(f_{k+1}|m)}{P(f_{k+1}|n)} \ .$$

Now let the useful signal have an unknown parameter T which is to be measured; i.e., $m(t) = m(t,T)$ and T varies continuously. Then from the following likelihood relations, where $P(T)$ is the a priori probability that T is present,

$$L(f_{k+1}|T) = P(T)\frac{P(f_{k+1}|T)}{P(f_{k+1}|n)} , \text{ and } L(f_{k+1}) = \int L(f_{k+1}|T)dT ,$$

we get, after substituting where appropriate, that

$$L(f_{k+1}) = \frac{\int P(f_{k+1}|T)P(T)dT}{P(f_{k+1}|n)} .$$

When the input signal f_{k+1} is being examined by the detector, the signals $f_1, f_2, \cdots, f_k$ have already been examined. We desire to make use of the results of those examinations and to do so in an optimum manner. The Bayes optimum way to use the prior inputs $f_1, f_2, \ldots, f_{k-1}$ is to compute a likelihood ratio based on these prior inputs,

$$L(f_k|f_1,f_2,\cdots,f_{k-1}) = \int L(f_k|T)P(T|f_1,f_2,\cdots,f_{k-1})dT$$

where any $f_i, i < k$, may or may not have had a useful signal component $m(t,T)$. Fralick [1] shows that a recursive relationship may be derived for $P_k(T)$. In Appendix A it is shown that $P_k(T)$ is related to $P_{k-1}(T)$ by the following recursive relationship,

$$P_k(T) = P_{k-1}(T)\frac{L(f_k|T) + \alpha}{L_{k-1}(f_k) + \alpha} , \text{ where } \alpha = \frac{1 - P(T)}{P(T)} ,$$

$P(T)$ is the a priori probability that the parameter T is present. This recursive relation may be considered to be the result of a delay-feedback loop or learning loop in which the value $P_{k-1}(T)$ serves as a memory of past observations. A diagram of the information flow which creates this loop is shown in Figure 1.

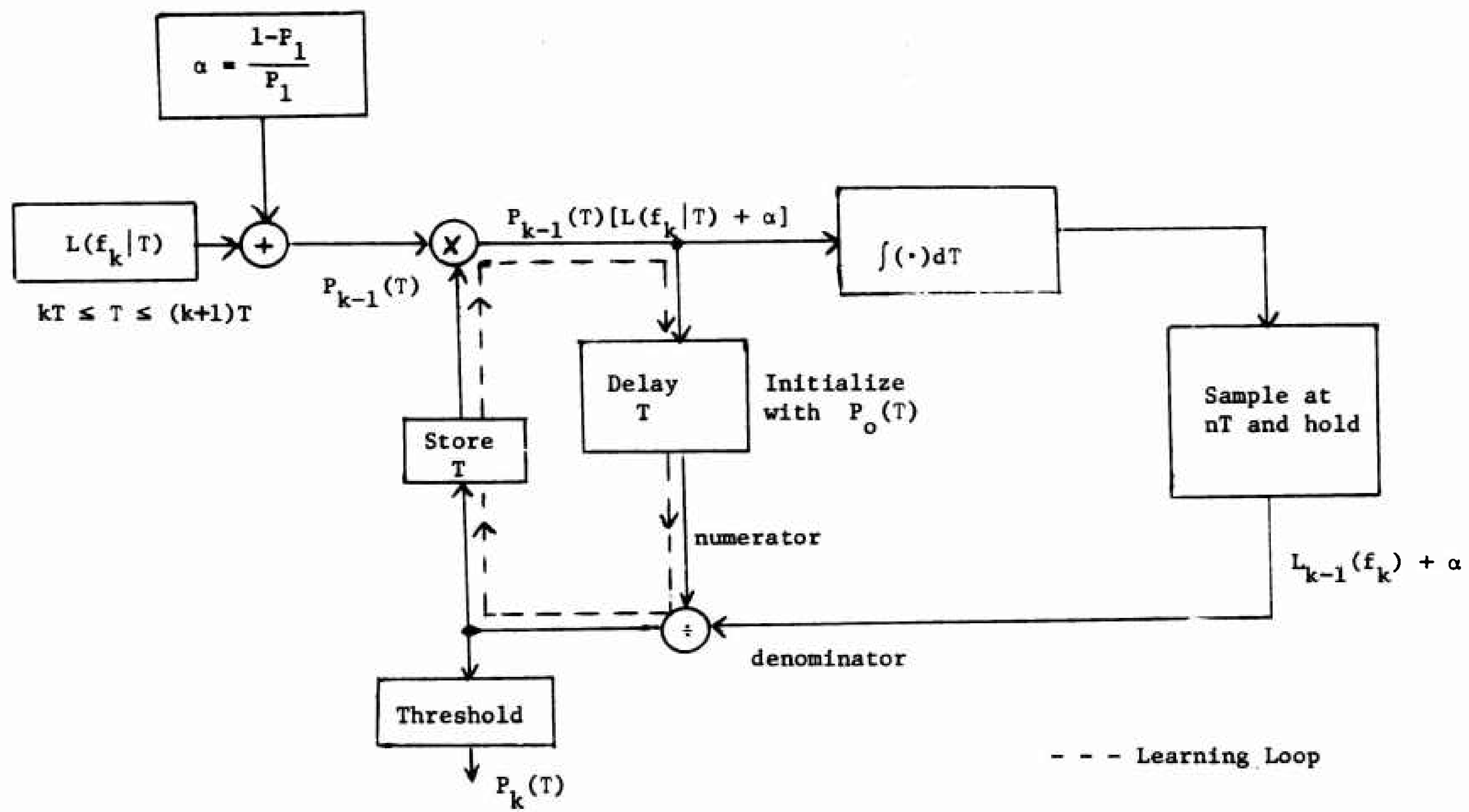

Figure 1. Adaptive Decision Process

The operation of the physical realization of the adaptive decision process is straightforward.

1. The input to the process is obtained from a likelihood ratio computer capable of providing $L(f_k|T)$ for each possible value of T in the range being scanned.
2. The values of $P_{k-1}(T)$ for each value of T are computed in the learning loop. The learning loop is initialized with a value $P_o(T)$, the a priori probability that a signal with the parameter is present when the detection process is started.
3. As the values of $[L(f_k|T) + \alpha]$ and $P_{k-1}(T)$ become available, they are multiplied and the product then integrated.
4. Once each T seconds $P_{k-1}(T)$ is sampled and compared against the threshold to determine if the useful signal is present or not.

Fralick [1] discusses the application of an adaptive decision process to the design of a radio receiver for use in locating electromagnetic signals with known amplitude and phase characteristics which are transmitted (for long periods of time) on an unknown frequency. Typical results were that the signal was identified by the receiver in ten sweeps across a frequency band in which the signal was known to be located with a signal-to-noise ratio of -11 dB and 100 sweeps when the signal-to-noise ratio was lowered to -17 dB. It is noted that the unique signal of interest must be on the air for a length of time sufficiently long to allow multiple looks throughout the frequency band being investigated and further that the receiver model is responsive to only one useful signal in the frequency band being scanned.

V. OPTIMUM CHARACTERISTICS OF THE ADAPTIVE PROCESS

Wainstein and Zubakov [2, sec. 57] discuss the problem of detecting a signal with an unknown parameter and combining measurement of the parameter with detection. They view the problem as one which is resolved by using multichannel receivers (detectors); i.e., the band of frequencies containing the carrier frequency of an electromagnetic signal in this case may be quantized and each channel tuned to a separate portion of the frequency band. Multichannel receivers may be divided into three catgories depending on how the detection and measurement process is performed:

1. Type I receiver first carries out detection and then measures the parameter.
2. Type II receiver carries out detection and measurement in parallel.
3. Type III receiver first measures the parameter, expressed as a likelihood ratio, and then uses the likelihood ratio to verify the presence of a signal with the measured parameter.

The multichannel Type I receiver is claimed by Wainstein and Zubakov [2] to be the optimum receiver for detection; it is optimum in the sense that it has minimum risk associated with the cost of decision errors. Fralick [1] shows that an adaptive (or learning) process may be employed on the multichannel Type I receiver provided the prior probabilities, that the parameter of interest is on a particular channel, are independent. The learning process updates these probabilities on each channel in the recursive fashion

previously described; i.e. the value of $P_{k-1}(T)$ is modified. When learning is completed, all the channel probabilities will have gone to zero except for the channel that contains the parameter of interest; that channel will have a value of $P_{k-1}(T) > 0$. The effect of the recursive updating is to adapt the multichannel Type I receiver to what is effectively a single-channel receiver.

If the input to the detection system can be scanned serially, one single-channel learning receiver may be used as a detection receiver instead of the multichannel receiver which operates in a parallel scanning mode. As has been noted previously, this restricts the application of the learning process to a single event which occurs frequently in the interval we can observe or which is present long enough to be scanned.

VI. MEASURES OF EFFECTIVENESS

The detector (receiver) which employs learning on serially inputed data has, initially, a detection and measurement performance capability equivalent to the Type I multichannel receiver as described previously. As the detector adapts to the parameter of interest through its recursive learning process, its performance is approaching, and finally reaches, that of a single-channel narrow-band receiver; which has the same effect in improving signal detection capability as would an increase in the sensitivity of each channel of the multichannel receiver. With an adaptive detector a lower signal-to-noise ratio (μ) is thus required to achieve given values of correct detection probability (D) and false alarm probability (F) than would be required with a multichannel receiver with the same sensitivity.

Wainstein and Zubakov [2, p. 299] show that for given values of F and D, where F is small, the threshold signal-to-noise ratio for a multichannel receiver with M channels is

$$\mu_M = \frac{\ln\frac{1}{F} + \ln M}{\ln\frac{1}{D}} - 1 \quad .$$

For a single-channel narrow band receiver such as the learning receiver working on serially inputed data, the threshold signal-to-noise ratio required to achieve the same level of detection probability and false alarm probability is,

$$\mu_L = \frac{\ln\frac{1}{F}}{\ln\frac{1}{D}} - 1 \quad .$$

The ratio μ_M/μ_L may be used as a measure of the improvement in sensitivity which would be expected when a recursive learning device is being considered as a substitute for a multichannel device without the learning feature.

The time that is required for the adaptive detector to make the transition from a device with a multichannel capability to a device with single-channel characteristics is a second measure of effectiveness. Fralick, et. al., [1] used simulation techniques to determine the transition time for various signal-to-noise ratios; the observation was made that closed-form expressions for the transition times were not known.

Wainstein and Zubakov [2, p. 361] note that for the situation when F and D are specified and the observation time is left as a random variable, the optimum decision rule is the Wald sequential probability ratio test (SPRT) with the thresholds L^* and L_*

given by the formulas

$$L^* = \frac{1-D}{1-F}\,, \quad \text{and}$$

$$L_* = \frac{F}{D}$$

Recall from previous discussion that

$$\mu_L \ln D = \ln\frac{F}{D}\,,$$

where μ_L is the threshold signal-to-noise ratio for a given F and D, when F is small. In view of the relation of the ratio F/D to both the lower threshold of the SPRT and to μ_L, it is proposed that the characteristics of the expected number of observations made in a SPRT, before a decision is reached, be used to develop a measure of the transition time for an adaptive detector to attain the level of performance of a single-channel receiver.

VII. DETERMINATION OF TRANSITION TIME

The proposed use of the expected number of observations made in a SPRT as a means of estimating transition time is based on the similarity between the SPRT and the adaptive decision process. First, there is the relation of the ratio F/D to both the lower threshold of the SPRT and to μ_L. Second, there is the quantitative demonstration in [1] that the adaptive decision process terminates in a finite number of observations provided the threshold μ_L is exceeded by the useful signal; it is well known that the SPRT, with probability 1, will yield a decision after a finite number of observations.

Based on this tenuous similarity between the thresholds and observation characteristics of two decision processes, the expected transition time which will be developed here is only a weak estimate

of the true expected transition time. It is strongly suspected, though, that it will be a conservative estimate in that it will overestimate the transition times.

Following Sverdrup [3], the derivation of the expected number of observations in a SPRT is developed in Appendix B. From that development we get the expected number of observations $E(N)$ to be

$$E(N) = \frac{D \ln\frac{F}{D} - (1-D)\ln\frac{1-D}{1-F}}{E(Z)} \text{ , where}$$

$$Z = \ln\frac{g(f|m)}{g(f|n)} \quad \text{and} \quad Z \neq 0 \ .$$

VIII. COMPUTATION OF E(N)

As an example of the procedure for determining the expected number of observations, $E(N)$ in a signal detection problem, we will apply the relation derived in Appendix B to the adaptive receiver process described in [1] and compute the number of observations which would be expected for the receiver parameters described in [1].

The adaptive receiver described in [1] operated with a false alarm probability $F = 10^{-4}$ and a correct detection probability $D = 0.5$. The adaptive receiver was used to determine the frequency of a sinusoidal signal with known parameters of amplitude and phase. Wainstein and Zubakov [2, p. 174] show that the likelihood ratio relating to this set of conditions has the form,

$$\frac{g(f|m)}{g(f|n)} = \frac{1}{1+\mu} \exp\left[\frac{Q^2}{2(1+\mu)}\right] \ ,$$

where μ is the effective signal-to-noise ratio and Q^2 is a parameter which describes the envelope of the signal of interest.

After taking the natural logarithm of the likelihood ratio we get

$$Z = \ln \frac{g(f|m)}{g(f|n)} = \ln \frac{1}{1+\mu} + \frac{Q^2}{2(1+\mu)} \quad ,$$

which has an expected value,

$$E(Z) = \ln \frac{1}{1+\mu} + E\left(\frac{Q^2}{2(1+\mu)}\right) \quad .$$

Evaluation of $E(Q^2/2(1+\mu))$ could be quite difficult if the exact nature of the envelope (Q) were not known; substitution eliminates this problem. Wainstein and Zubakov [2, p. 176] discuss the use of the parameter Q^2 in a simple detection decision rule:

If $Q^2 \geq Q_*^2$, decide that $f(t) = m(t) + nt$;

If $Q^2 < Q_*^2$, decide that $f(t) = n(t)$,

where Q_*^2 is the decision threshold. This threshold is shown in [2] to be related to the probability of detection (D) in the following manner,

$$D = \exp\left(\frac{-Q_*^2}{2\mu(1+\mu)}\right) \quad .$$

After taking the natural logarithm of this expression we have the following relation between D and Q_*^2 ,

$$-\mu \ln D = \frac{Q_*^2}{2(1+\mu)} \quad .$$

Now, Q^2 must be at least equal to Q_*^2 , if not greater than Q_*^2 , in order to make the decision that $f(t) = m(t) + n(t)$. We may therefore substitute Q_*^2 for Q^2 in the expression for $E(Z)$ on the assumption that the signal of interest, $m(t)$, is present. After this substitution, we then have

$$E(Z) = \ln \frac{1}{1+\mu} - \mu \ln D.$$

With this result, the expected number of observations to be made by the adaptive receiver in order to detect the signal of interest is seen to be

$$E(N) = \frac{D\ln\frac{F}{D} - (1-D)\ln\frac{1-D}{1-F}}{\ln\frac{1}{1+\mu} - \mu\ln D} .$$

If the operating parameters, $D = 0.5$ and $F = 10^{-4}$, are substituted into the formula for $E(N)$, we find that approximately 180 observations are expected at a signal-to-noise level of -11 dB ($\mu = 0.08$) and approximately 600 observations are expected at a signal-to noise level of -17 dB ($\mu = 0.02$). As was anticipated, these numbers are significantly greater than the actual numbers of observations needed to detect the signal of interest as reported by Fralick _et_. _al_. [1]. The disparity between the computed number of expected observations and the actual results reported in [1] is assumed to be influenced by two characteristics of the adaptive decision process:

1. A strong _a priori_ opinion that the signal of interest will in fact be scanned (i.e. a value of $P_o(T) = 0.75$ for example) would be a factor in reducing the number of scans. Loosely speaking, the recursive effect of the likelihood ratios would bring the value of $P_k(T)$ to the acceptance threshold with fewer scans with a larger starting value of $P_o(T)$ than with a small value of $P_o(T)$; provided, of course, the signal is in fact there. If the signal is not present, a strong _a priori_ opinion that it is present would be expected to require more scans to determine that it, in fact, is not present.

In the results described in [1] the signal of interest was always present; the value of $P_0(T)$ used in the receiver was not reported. The SPRT is not influenced by any *a priori* opinion regarding the state of the set of signals being scanned; the same number of scans would be expected whether the signal was present or not.

2. The adaptive decision process is designed to take advantage of the accumulation of knowledge about the presence, or absence, of the signal of interest. The SPRT, on the other hand, is not designed to take advantage of trends and one contrary observation is sufficient to totally disrupt a trend towards one or the other of the decision thresholds.

IX. APPLICATIONS

In the preceding discussion of the development and application of an adaptive decision process, the emphasis has been on the application of an adaptive procedure employing multiple observations to the special problem of optimizing electromagnetic signal detection. In addition to that useful application, the adaptive decision model, which relies heavily on a learning process, may be used to study other, more general, situations in which multiple observations are employed in formulating a detection decision.

Multiple observations are typically encounterered in two types of situations:

1. The first is the generalization of the situation discussed in this thesis; the situation in which observations are

accumulated and the total weight of the evidence is used as the basis for a decision regarding the presence or absence of a useful signal.

2. The second is the situation in which a decision is made based on multiple, independent observations. The detection decision rule usually employed is that a useful signal is present if any one of the multiple observations indicates the presence of a useful signal.

As an example of the application of the adaptive decision process developed in this thesis we will consider two situations in which humans act as signal detectors. The first situation is a straightforward comparison of multiple, independent observers to a multi-channel receiver; the second is an example of the recursive learning process acting against detection.

A. MULTIPLE OBSERVERS

A common practice in practical detection problems is to use a team of observers who are to act independently and attempt to detect some signal of interest. The rationale for this practice is that the group effort is expected to improve the chances of detecting the signal of interest. For example, if two independent observers have an individual probability of detection $D = 0.5$, then the probability that one or the other or both will detect the signal is 0.75.

Green and Swets [4, p. 248] discuss psychometric studies conducted to test whether or not teams of observers would show this degree of improvement in detection capabilities. The typical finding in the studies was that if a gain in detection capability was made it was

not significant and further that an increase in the proportion of false alarms was experienced when the size of the team increased.

Recalling the relation for the threshold signal-to-noise ratio for a multichannel receiver and rearranging terms, we have a relation between detection parameters which is seen to apply to the team situation; i.e.,

$$\mu_M = \frac{\ln \frac{1}{F}}{\ln \frac{1}{D}} - 1 + \frac{\ln M}{\ln \frac{1}{D}} ,$$

where M is the number of observers on the team. We see that the larger the team gets, the larger the value of F which must be accepted if the threshold signal-to-noise ratio is fixed and the detection probability (D) remains constant. If it were possible to hold F at a fixed value while M is increased, the model predicts that D would decrease if the signal-to-noise ratio is fixed.

A possible solution to the false alarm problem is to replace a team of observers by a single observer with the capability of employing the adaptive decision process. This action would have the benefits shown for the adaptive receiver when it replaced the multichannel receiver, if the human observer is at least as predictably "optimum" as he is predictabley "faulty". Alternatively the team could report to a decision maker who would be guided by the adaptive decision procedures.

B. VIGILANCE

The adaptive decision process model may also be applied to the detection problem characterized by extended periods of observation

during which signals can occur at any time without warning and with no predictive spacing between signals. This problem is referred to as the "vigilance" problem. In this type of signal detection environment there is a known deterioration in detection capability as the observation period proceeds.

The effect in these situations is as if the recursive learning process were being reversed. That is, the span (channels) of attention widens and the learning process may be considered to be strengthening an a priori opinion that no signal will be encountered. Green and Swets [4, p. 332] discuss the vigilance problem and it is again interesting to note that human performance seems to be following the characteristics of an adaptive receiver model.

Limited studies on the vigilance problem indicate that the probability of detection decreases over time; interestingly, the false alarm probability has also been observed to decrease. Looking again at the relation for μ_M, we see that as the span of attention widens, the value of the term $\ln M/\ln\frac{1}{D}$ tends to increase. If a fixed value of μ_M is assumed, the model predicts that D must decrease to maintain the equality of the relationship; further, the term $\ln\frac{1}{F}/\ln\frac{1}{D}$ suggests that if some directly proportional relationship were to exist between F and D then F would also decrease.

A possible solution to the vigilance problem is suggested by the recursive learning feature of the adaptive receiver. To apply this solution it must be assumed that the probability of detection (D) by a human observer is maintained at an acceptable level by focusing the observer's attention. Then, since the recursive learning process is strongly influenced by the likelihood that a useful signal is

present during the scan across the signal environment, it may be possible to focus attention by introducing dummy useful signals into the detection process.

X. CONCLUSION

When multiple observations are available, or are required, for use in the solution of a detection problem, the optimum utilization of these observations is through an adaptive decision process. In comparison to a simple detection process based on single independent observations, the adaptive decision process enables the observer to make a detection decision with the same level of detection error at a smaller signal-to-noise ratio (μ) or conversely for a fixed μ and fixed false alarm probability a higher probability of correct detection is possible through the adaptive process.

The adaptive decision process is limited to signals of long duration due to the nature of the learning process used in this decision process. A procedure for computing the expected length of the decision process is developed. Based on this expected length of time to complete the decision process, the observer may evaluate the suitability of employing a relatively lengthy decision process with optimum properties to solve a detection problem.

Lastly, the learning cycle used in the adaptive decision process suggests that the model of an adaptive decision process may be used to predict the behavior of human observers performing signal detection functions. Two such situations were discussed and it was shown that the adaptive decision process model does in fact lend itself to analyzing the behavior of human observers.

APPENDIX A: DERIVATION OF RECURSIVE FORM

When the input signal $f_{k+1}(t)$ is being examined by the detector, the signals $f_1, f_2, \cdots, f_k$ have already been examined. The Bayes optimum use of the prior inputs $f_1, f_2, \cdots, f_{k-1}$ is

$$L(f_k|f_1,f_2,\cdots,f_{k-1}) = \int L(F_k|T)P(T|f_1,f_2,\cdots,f_{k-1})dT$$

where

$$f(t) = m(t,a,T,\theta) + n(t); \text{ and}$$

the useful portion $m(t,a,T,\theta)$ consists of a counting parameter, such as time, $\underline{t}$; known parameters $\underline{a}$; unknown parameters $\underline{T}$ which are measured upon detection; and unknown parameters $\underline{\theta}$ which are not measured. The purpose of examining $f(t)$ is to learn the nature of $\underline{T}$, for example to find the frequency on which a radio signal of known characteristics is being transmitted.

Consider the probability density $P(T|f_1,f_2,\cdots,f_k)$. By the Bayes formula, it may be expressed as

$$P(T|f_1,f_2,\cdots,f_k) = \tag{1}$$

$$\frac{P(f_k|T,f_1,f_2,\cdots,f_{k-1})P(T|f_1,f_2,\cdots.f_{k-1})}{P(f_k|f_1,f_2,\cdots,f_{k-1})} .$$

Under the assumption that each f_i is independent of all other f_j, we may further state that

$$P(f_k|T,f_1,f_2,\cdots,f_{k-1}) = P(f_k|T) .$$

Expressing $P(f_k|T)$ as a compound event gives

$$P(f_k|T) = p_1P(f_k|T,m(t,T)) + (1-p_1)P(f_k|n(t))$$

where p_1 is the *a priori* probability that

$$f(t) = m(t) + n(t)$$

and $1-p_1$, is the a priori probability that

$$f(t) = n(t) \quad .$$

Next, after factoring $p_1 P(f_k|n(t))$ we have

$$P(f_k|T) = p_1 P(f_k|n(t)) \left[L(f_k|T\ ) + \frac{1-p_1}{p_1} \right] . \tag{2}$$

Next, by integrating equation (1) over all possible values of T we get

$$P(f_k|f_1,f_2,\cdots,f_{k-1}) = \int P(f_k|T)P(T|f_1,f_2,\cdots,f_{k-1})dT \ ,$$

which may be factored to give

$$P(f_k|f_1,f_2,\cdots,f_{k-1}) =$$

$$p_1 P(f_k|n(t)) \left[\int L(f_k|T)P(T|f_1,f_2,\cdots,f_{k-1})dT + \frac{1-p_1}{p_1} \right] . \tag{3}$$

After dividing (2) by (3) and cancelling common terms, we get

$$P(T|f_1,f_2,\cdots,f_k) =$$

$$P(T|f_1,f_2,\cdots,f_{k-1}) \frac{L(f_k|T)P(T|f_1,f_2,\cdots,f_{k-1}) + \dfrac{1-p_1}{p_1}}{\int L(f_k|T)P(T|f_1,f_2,\cdots,f_{k-1})dT + \dfrac{1-p_1}{p_1}} \ .$$

The recursive nature of this relation is more readily seen by defining two new quantities,

$$P_k(T) = P(T|f_1,f_2,\cdots,f_k) \quad \text{and}$$

$$L_{k-1}(f_k) = \int L(f_k|T)P_{k-1}(T)dT \quad .$$

Now the expression for $P_k(T)$ may be rewritten as

$$P_k(T) = P_{k-1}(T) \frac{L_k(f_k|T) + \alpha}{L_{k-1}(f_k) + \alpha} \ ,$$

$$\alpha = \frac{1-p_1}{p_1} \quad .$$

APPENDIX B: EXPECTED NUMBER OF OBSERVATIONS

In the sequential probability ratio test, the expected number of observations (N) is

$$E(N) = \sum_{m=1}^{\infty} mP(m|T) \text{ , where}$$

T is the unknown parameter of interest. It is convenient to introduce a variable Z_j which is defined as

$$Z_j = \ln \frac{f(X_j|T_1)}{f(X_j|T_o)} \text{ , where}$$

T_1 is the state in which the unknown parameter is present, T_o the state in which it is absent and to make use of the result, proved in [4], that

$$E\left[\sum_{j=1}^{N} Z_j\right] = E(N)E(Z) \quad .$$

Then provided $E(Z) \neq 0$, we can find $E(N)$. For this purpose we may write

$$E\left[\sum_{j=1}^{N} Z_j\right] = E\left[\sum_{j=1}^{N} Z_j|d_o\right] P(d_o) + E\left[\sum_{j=1}^{N} Z_j|d_1\right] P(d_1) \text{ ,}$$

where d_o is the decision made in the event T is not present and d_1 is the decision made in the event T is present. If the decision d_o is made, this implies

$$\sum_{j=1}^{N-1} Z_j > \ln \frac{F}{D} \text{ , and } \sum_{j=1}^{N} Z_j \leq \ln \frac{F}{D} \text{ .}$$

Now assuming that no single Z_j is likely to dominate in the sum we may say

$$E\left(\sum_{j=1}^{N} Z_j \middle| d_o\right) \approx \ln \frac{F}{D} ,$$

and similarly

$$E\left(\sum_{j=1}^{N} Z_j \middle| d_1\right) \approx \ln \frac{1-F}{1-D} .$$

After substituting, we then obtain

$$E\left(\sum_{j=1}^{N} Z_j\right) = P(d_o) \ln \frac{F}{D} + P(d_1) \ln \frac{1-F}{1-D} ,$$

from which the expected number of observations is seen to be

$$E(N) = \frac{P(d_o|T) \ln \frac{F}{D} + P(d_1|T) \ln \frac{1-F}{1-D}}{E(Z)}$$

since the probability is 1 that a decision is reached regarding the state of the unknown parameter T .

As an example of the computation technique for $E(N)$ consider a success or failure situation. Let T be the unknown fraction of failures in a sample of size N. Some action is considered worthwhile (d_o) if $T \leq T_o$ and not worthwhile (d_1) if $T \geq T_1$. If $T_o < T < T_1$ it is not important which decision is reached. The expression for $E(Z)$ is gotten from

$$Z = \ln \frac{f(X|T_1)}{f(X|T_o)} = \ln \frac{T_1^X (1-T_1)^{1-X}}{T_o^X (1-T_o)^{1-X}} ,$$

where X is the result of an inspection. Expanding the expression for Z gives

$$Z = \ln \frac{1-T_1}{1-T_o} + X \ln \frac{T_1(1-T_o)}{T_o(1-T_1)} , \text{ which yields}$$

$$E(Z) = \ln \frac{1-T_1}{1-T_o} + \ln \frac{T_1(1-T_o)}{T_o(1-T_1)} ,$$

when X is a defective. We now have

$$E(N) = \frac{P(d_o|T) \ln \frac{F}{D} + P(d_1|T) \ln \frac{1-F}{1-D}}{\ln \frac{1-T_1}{1-T_o} + \ln \frac{T_1(1-T_o)}{T_o(1-T_1)}} .$$

If $1 - T \approx 1$ and noting that $P(d_o|T) = D$ and $P(d_1|T) = 1 - D$, $E(N)$ may abe simplified as follows,

$$E(N) = \frac{D \ln \frac{F}{D} - (1-D) \ln \frac{1-D}{1-F}}{\ln \frac{1-T_1}{1-T_o}} .$$

LIST OF REFERENCES

1. Fralick. S.C., Slenkovick, F.L., and Wilson, D.L., "An Adaptive Receiver for Signals of Unknown Frequency", IEEE Trans. Communication Technology, Vol. COM-16, No. 5, pp. 648-656, October 1968.

2. Wainstein, L.A. and Zubakov, V.D., Extraction of Signals from Noise, R.A. Silverman, Transl., Prentice-Hall, 1962.

3. Sverdrup, E., Laws and Chance Variations, Vol II, Rand McNally, 1967.

4. Green, D.M. and Swets, J.A., Signal Detection Theory and Psychophysics, Wiley, 1966.

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

An Adaptive Decision Process

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; December 1969

5. AUTHOR(S) (First name, middle initial, last name)

Joseph Maris Moroney

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1969	33	4

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

This thesis investigates a decision process which is designed to employ multiple observations in an optimal manner as a means of solving a signal detection problem. The characteristic of this decision process is that it permits the decision-maker to defer his detection decision until new data is obtained from the next observation and to weigh the new data with an opinion based on previous data. The effect of adapting a decision to the results of previous observations is seen to be similar to a learning process which is taking place over a length of time. Since the decision process may involve relatively lengthy periods of time an estimator of this time is developed. Lastly, the decision model is seen to provide a model by which human detection behavior may be evaluated.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408